

# Exploring Many Flavor QCD with Wilson Fermion

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**N. Yamada (KEK/GUAS)**

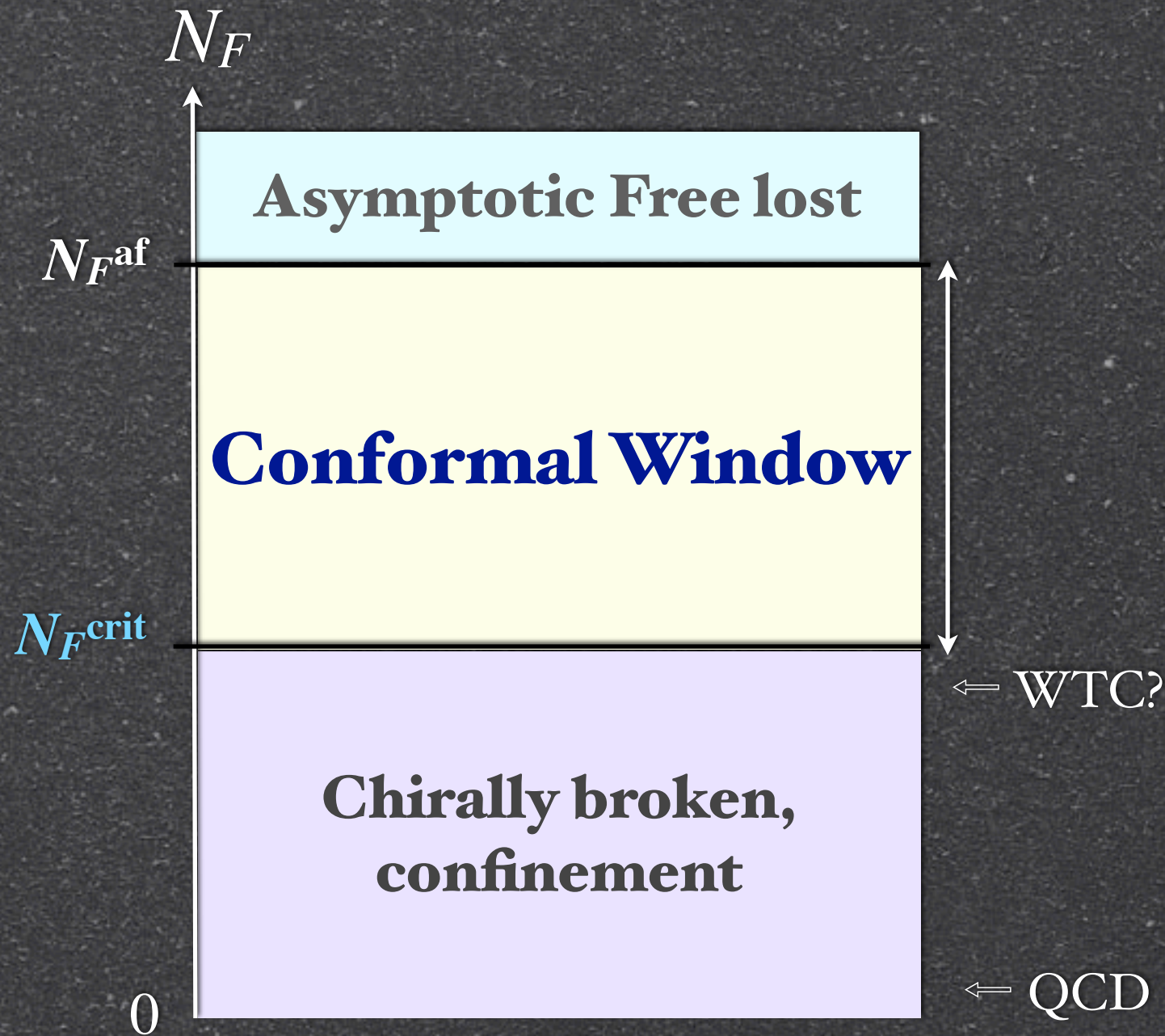
**based on collaboration with**

**M. Hayakawa (Nagoya), K.-I. Ishikawa (Hiroshima),  
Y. Iwasaki (KEK/Tsukuba), Y. Osaki (Hiroshima),  
S. Takeda (Kanazawa), S. Uno (Nagoya), T. Yoshie (Tsukuba)**



# Conformal Window

Phase of a theory with  $N_F$  degenerate massless flavors at  $T=0$



**Conformal Window:**

$$N_F^{\text{crit}} < N_F < N_F^{\text{af}}$$

Viable Technicolor model (e.g. WTC) is expected to exist in vicinity of  $N_F^{\text{crit}}$ .

**First task : Identifying  $N_F^{\text{crit}}$**

Phase diagram@ $T=0$



# We take Wilson fermion.

## Disadvantages:

- ✓  $O(a)$  scaling violation
- ✓ Fine-tuning,
- ✓ ...

## Advantages:

- ✓ Simple, tractable and well understood
- ✓ Able to study arbitrary  $N_F$  without any subtlety
- ✓ Independent check to KS (or other) results



# Contents

There are several approaches to identify  $N_F^{\text{crit}}$ .

Focusing on SU(3) gauge theory, we are performing the following studies:

1. Running coupling and anomalous dimension in 10-flavor QCD. IRFP?
2. Finite temperature study of Many Flavor QCD ( $N_F = 6 - 10$ )
  - Strategy
  - Future prospect



# $\alpha(\mu)$ and $\gamma_m$ in 10-flavor QCD

**M. Hayakawa, K.-I. Ishikawa, Y. Osaki,  
S. Takeda, S. Uno, NY**

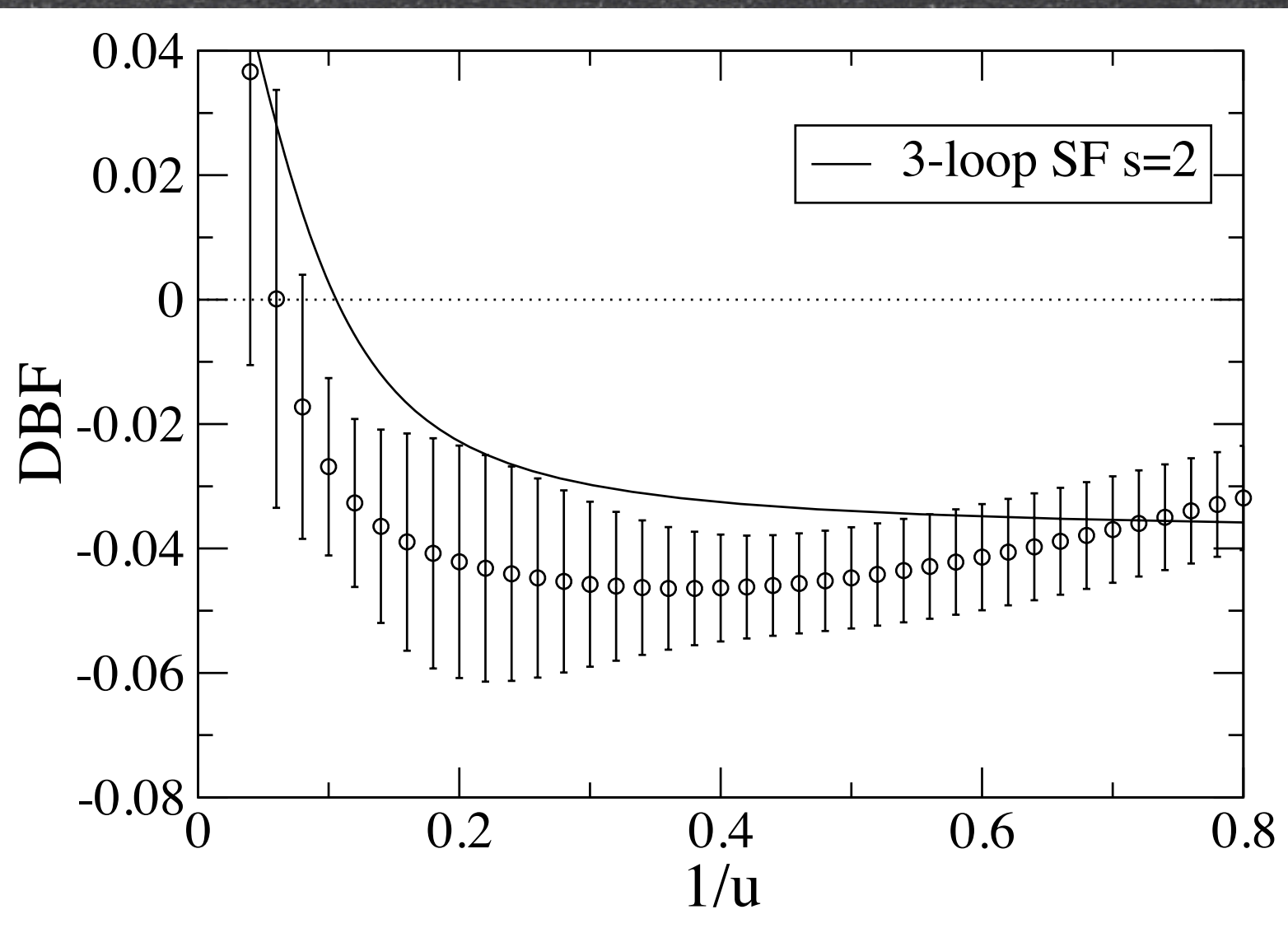


# $g_{\text{SF}}^2(\mu)$ in 10-flavor QCD

Hayakawa, Ishikawa, Osaki, Takeda, Uno, NY, PRD(2011) and work in progress

Preliminary

$$B^{\text{SF}}(u, s) = \frac{1}{g_{\text{SF}}^2(u, s)} - \frac{1}{u}$$



DBF=0  $\Rightarrow$  IRFP

$$g_{\text{FP}}^2 \geq 12$$

Continuum extrapolation  
with two data points.

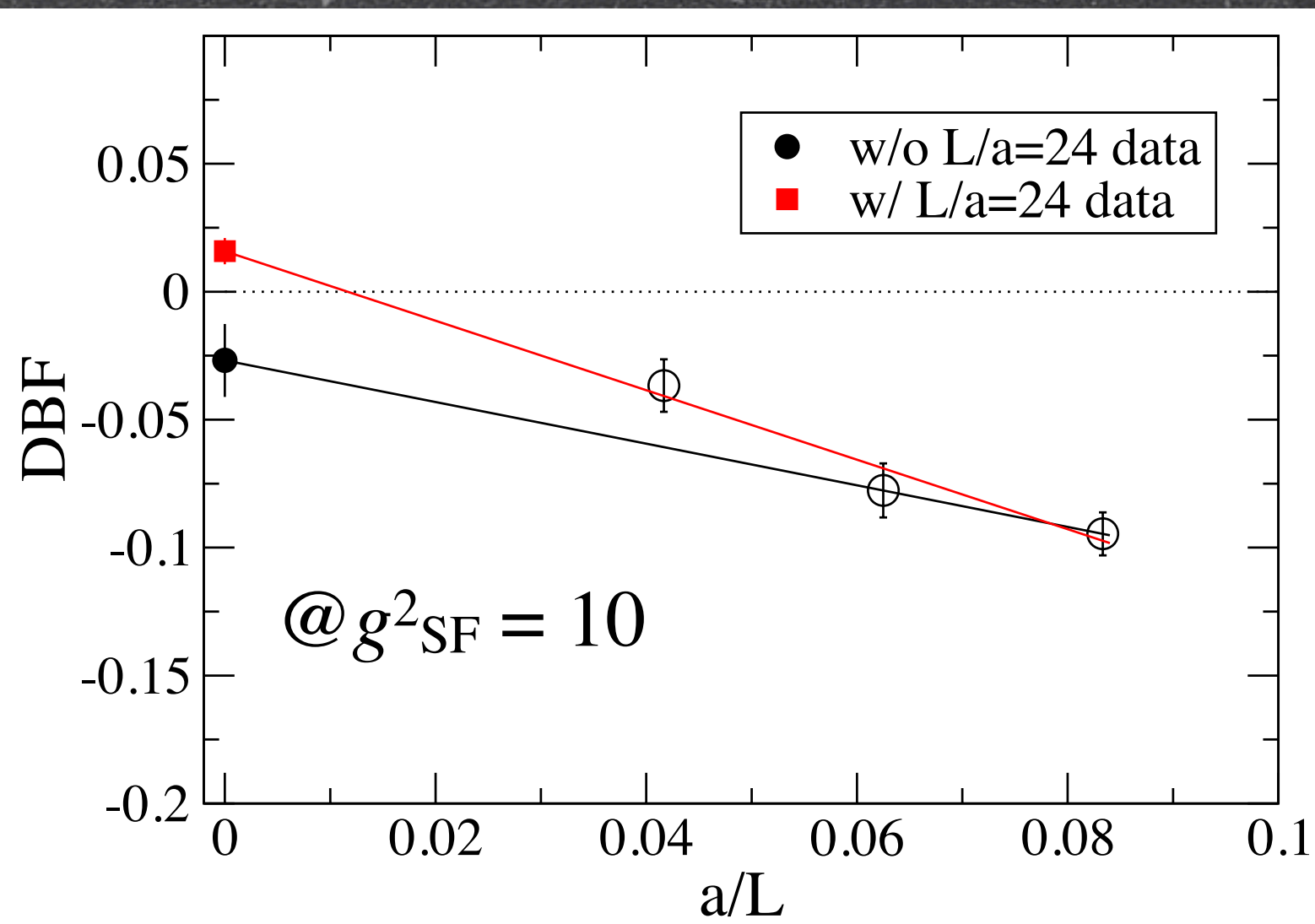
In order to have more  
confidence, large  $V$   
calculation is on-going.



# $g_{\text{SF}}^2(\mu)$ in 10-flavor QCD

Hayakawa, Ishikawa, Osaki, Takeda, Uno, NY, PRD(2011) and work in progress

Preliminary



Adding large  $V$  data, the continuum limit shifts upward.

$$g_{\text{FP}}^2 \geq 12 \Rightarrow g_{\text{FP}}^2 \sim 10$$



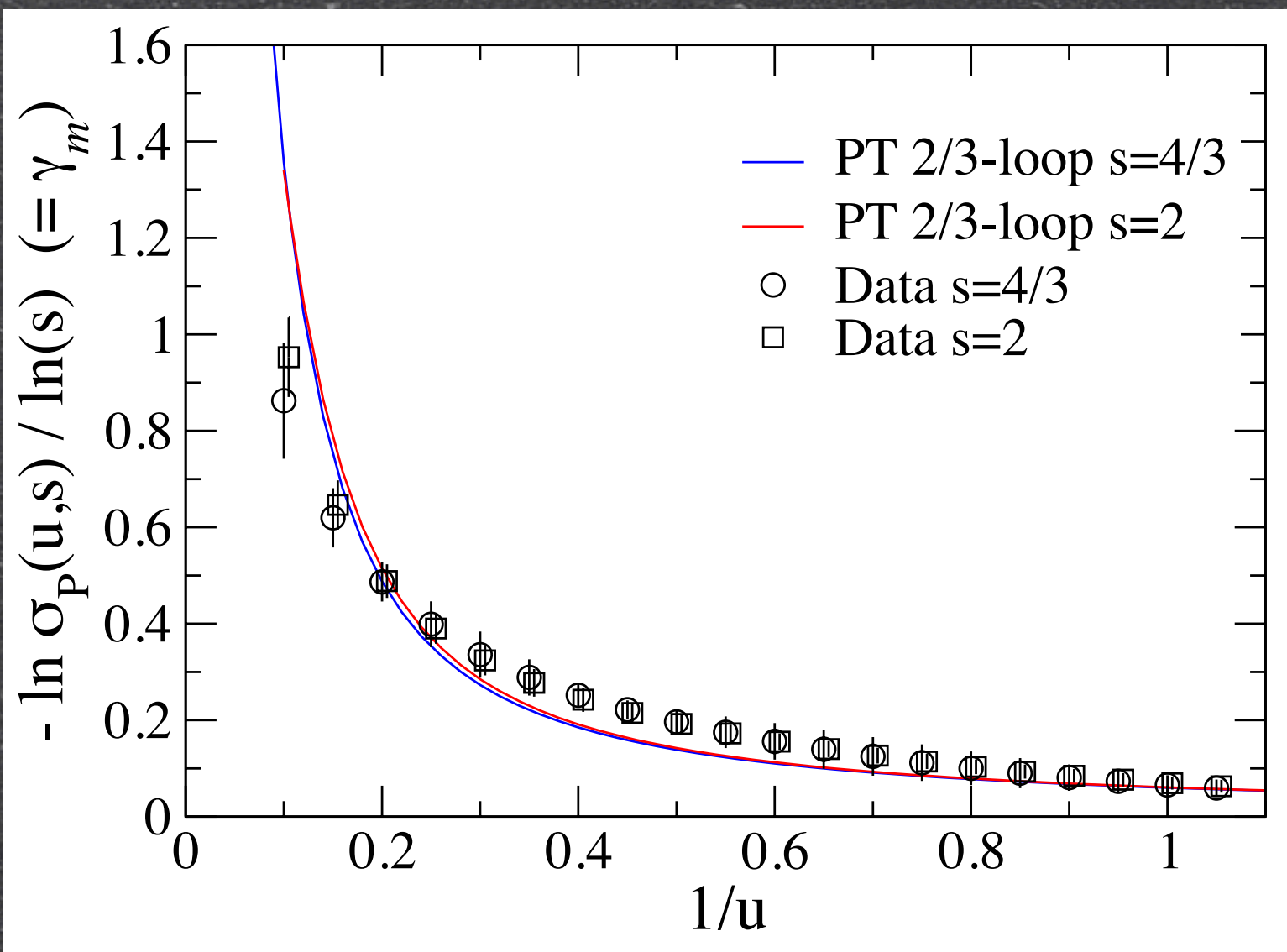
More confident on IRFP



# $\gamma_m$ of 10-flavor QCD

Hayakawa, Ishikawa, Osaki, Takeda, Uno, N.Y., work in progress

Preliminary



Two different step scaling factors give consistent result.

Assuming  $g_{FP}^2 \sim 10$ ,  $\gamma_m \sim 1$  !

**10-flavor QCD appears to be in CW ( $N_F^{\text{crit}} < 10$ ) and have  $\gamma_m \sim O(1)$ .**



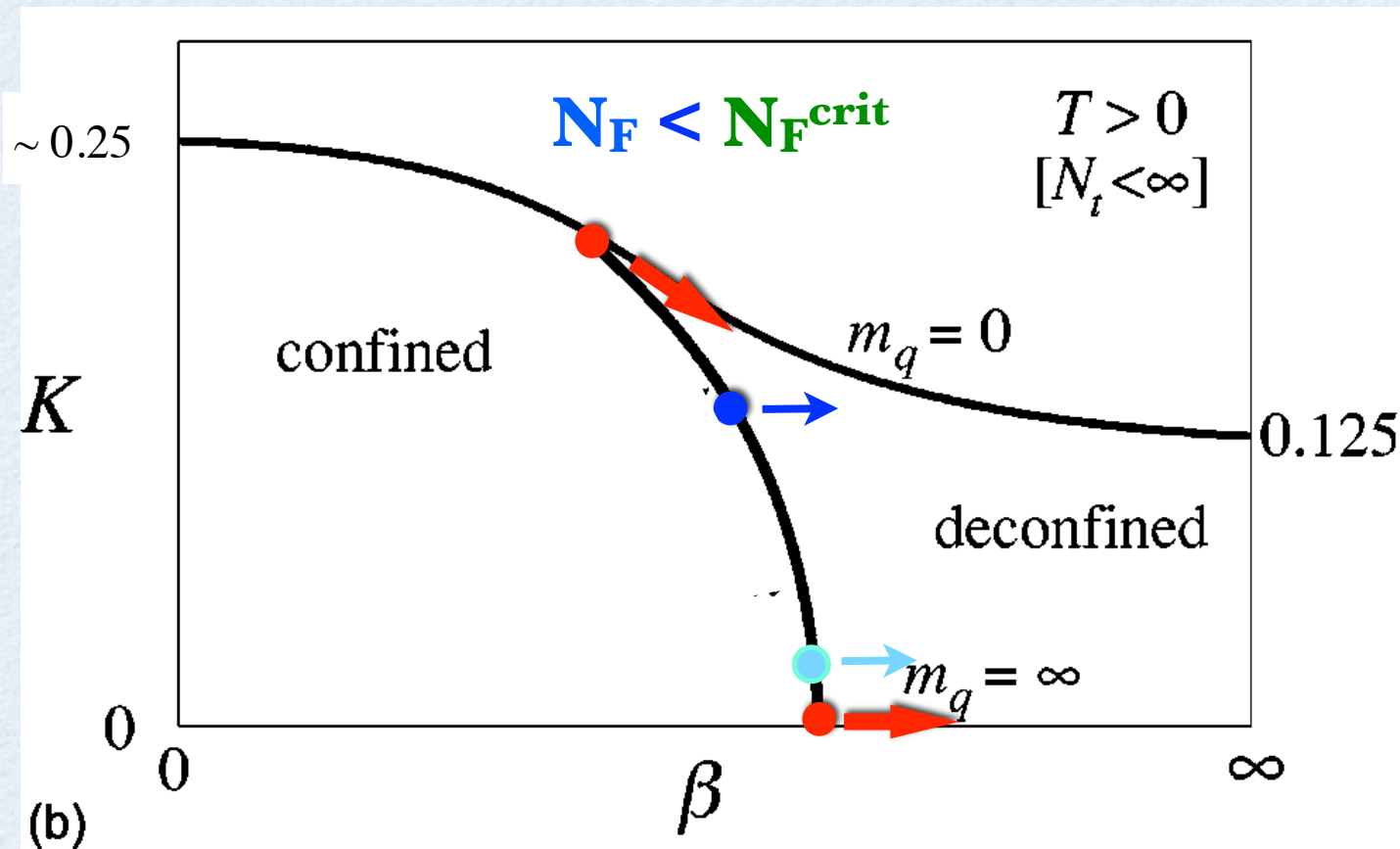
# Finite temperature study of Many Flavor QCD

**WMF Collaboration:** M. Hayakawa, K.-I. Ishikawa, Y. Iwasaki,  
S. Takeda, T. Yoshie, NY



# Phase diagram of Wilson fermion

for  $N_F < N_F^{\text{crit}}$  Iwasaki et al. (91,04)

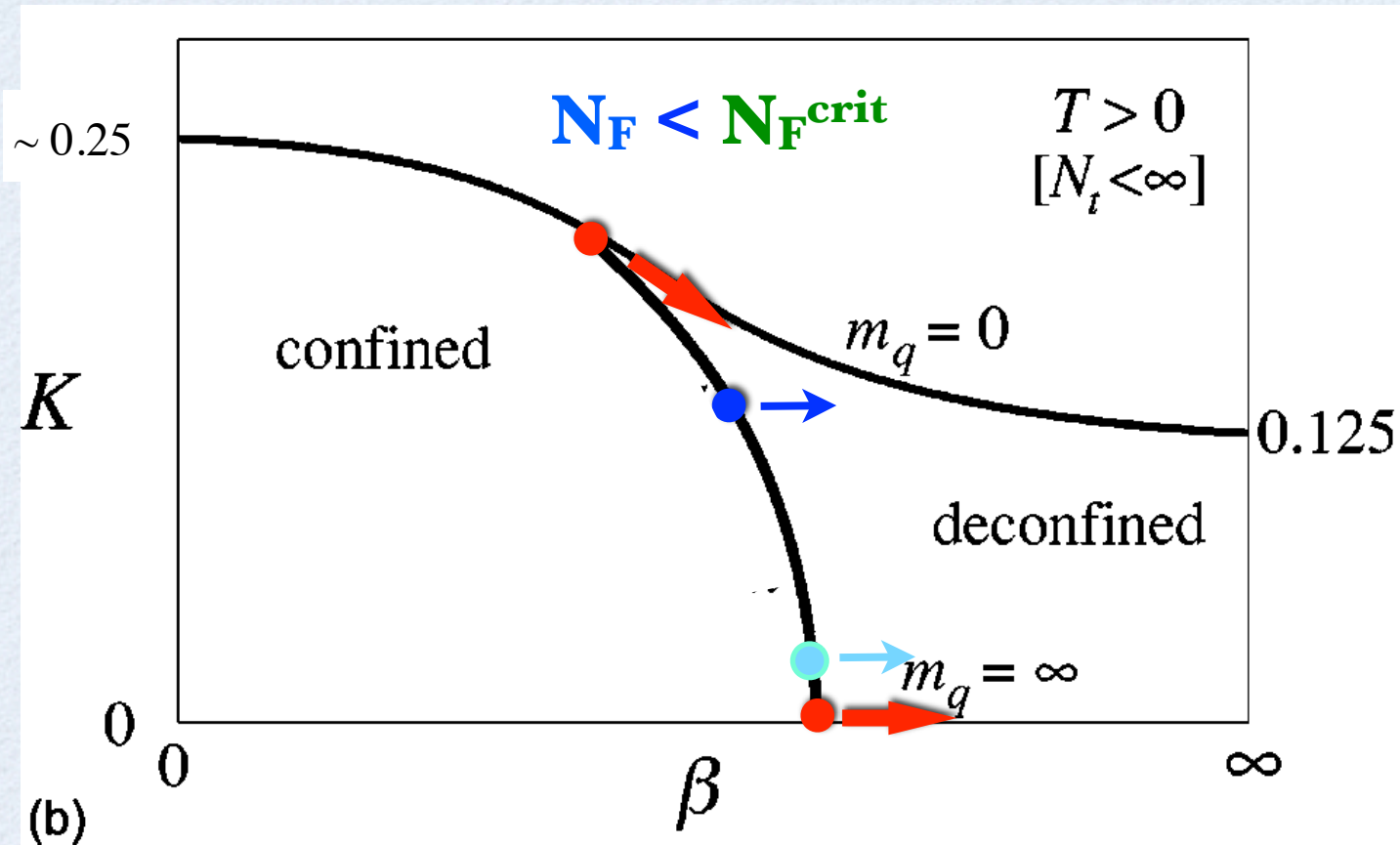


If the theory is confining, the transition line move to the right as  $T$  decreases (or  $V$  increases).



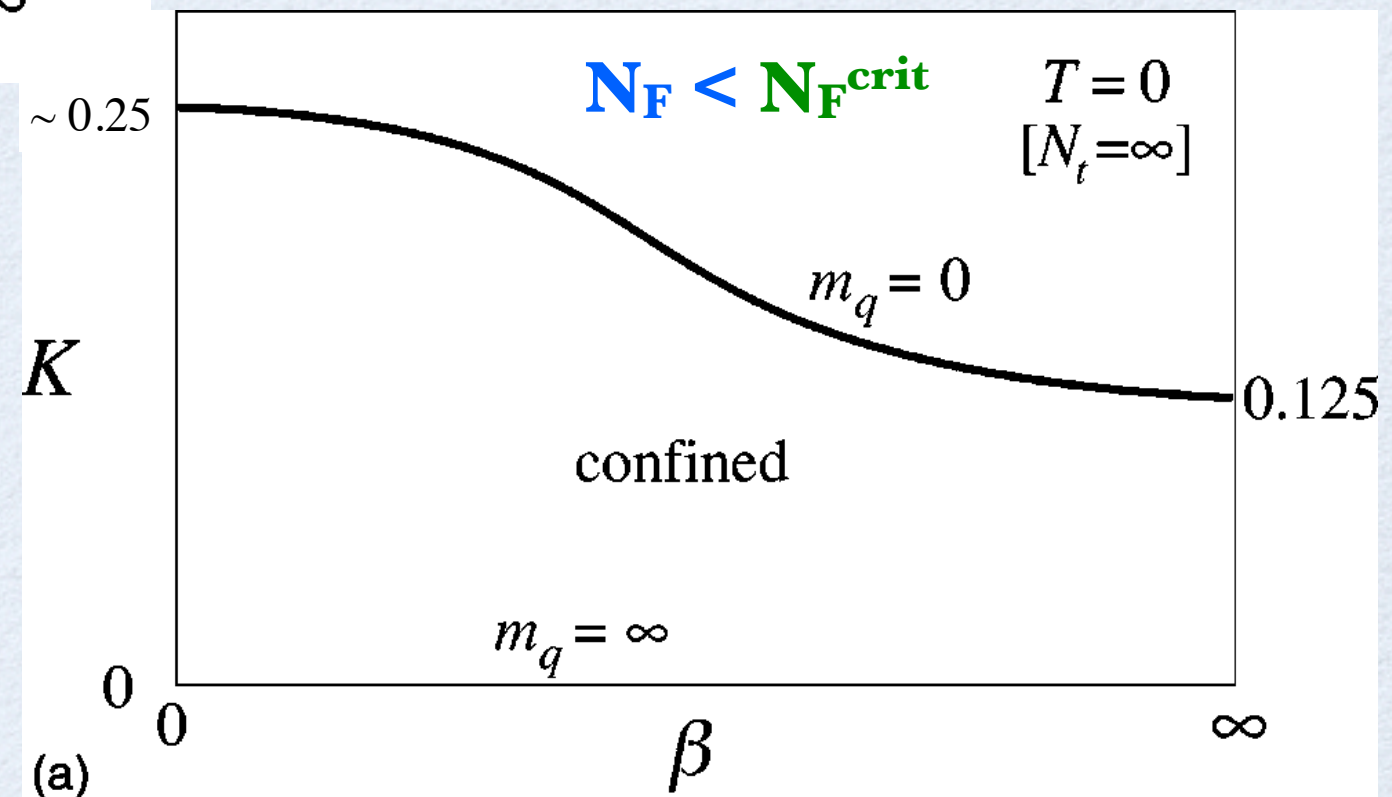
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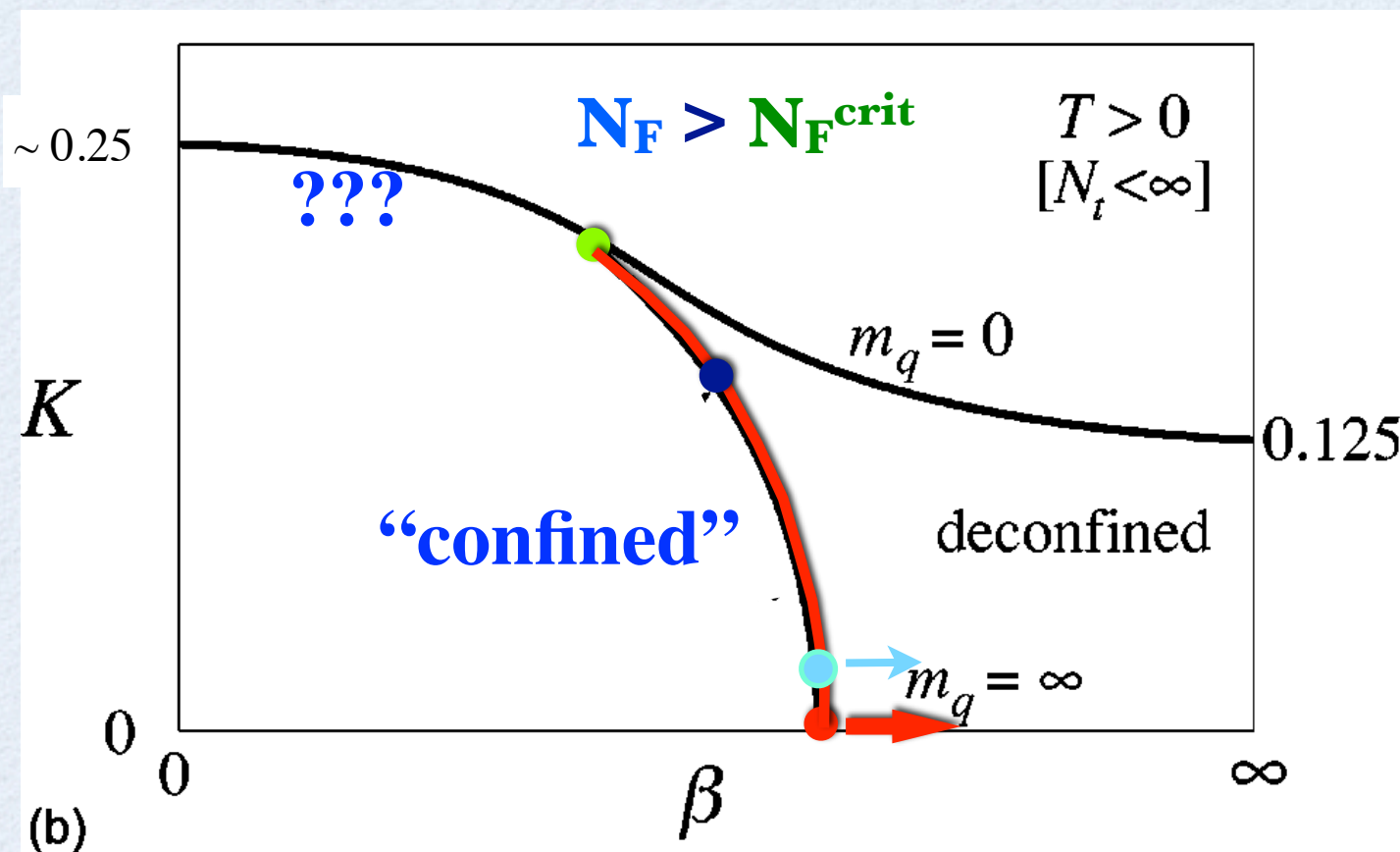
Eventually, the whole region is covered by confining phase.





# Phase diagram of Wilson fermion

with  $N_F^{\text{crit}} < N_F < N_F^{\text{af}}$  (speculation)

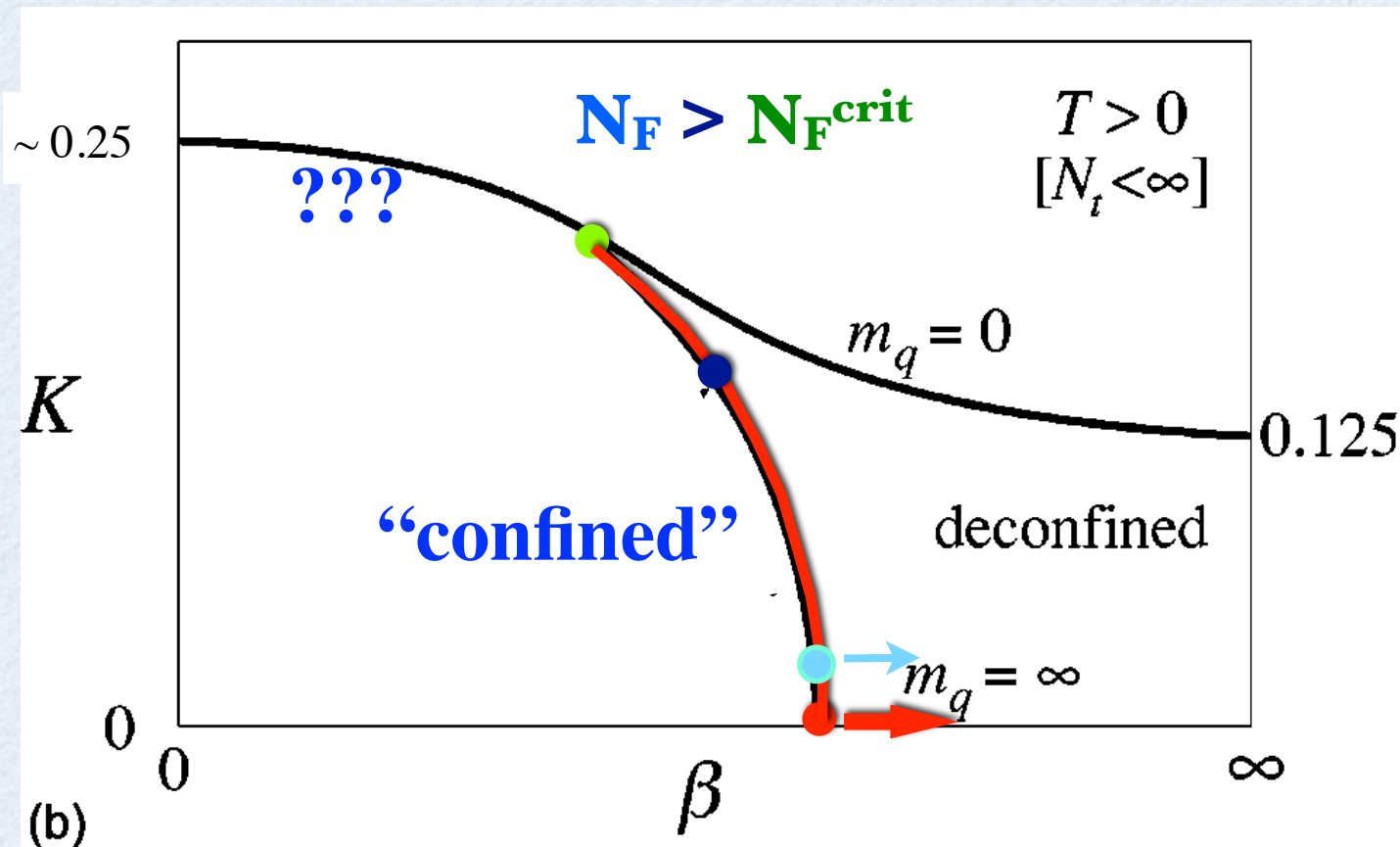


The one end of the transition line at  $m_q = \infty$  moves to the right with  $V$  as before, while the other end at  $m_q = 0$  won't.



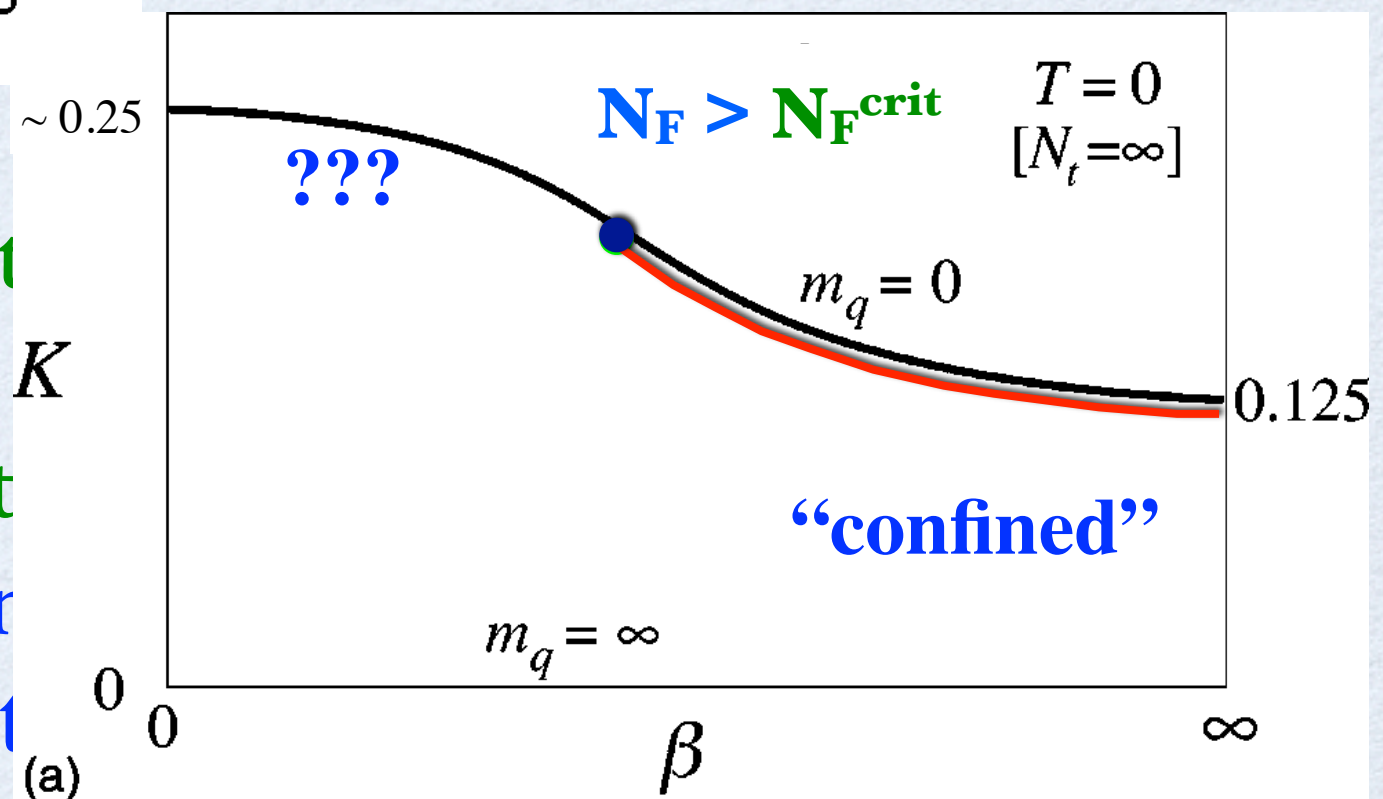
# Phase diagram of Wilson fermion

with  $N_F^{\text{crit}} < N_F < N_F^{\text{af}}$  (speculation)



The one end of the transition line at  $m_q = \infty$  moves to the right with  $V$  as before, while the other end at  $m_q = 0$  won't.

In the large  $V$  limit, the relevant region is covered by “confined” phase except for the chiral limit. In “confined”,  $\Lambda_{\text{TC}}$  depends on  $m_q$  and vanishes in the chiral limit.





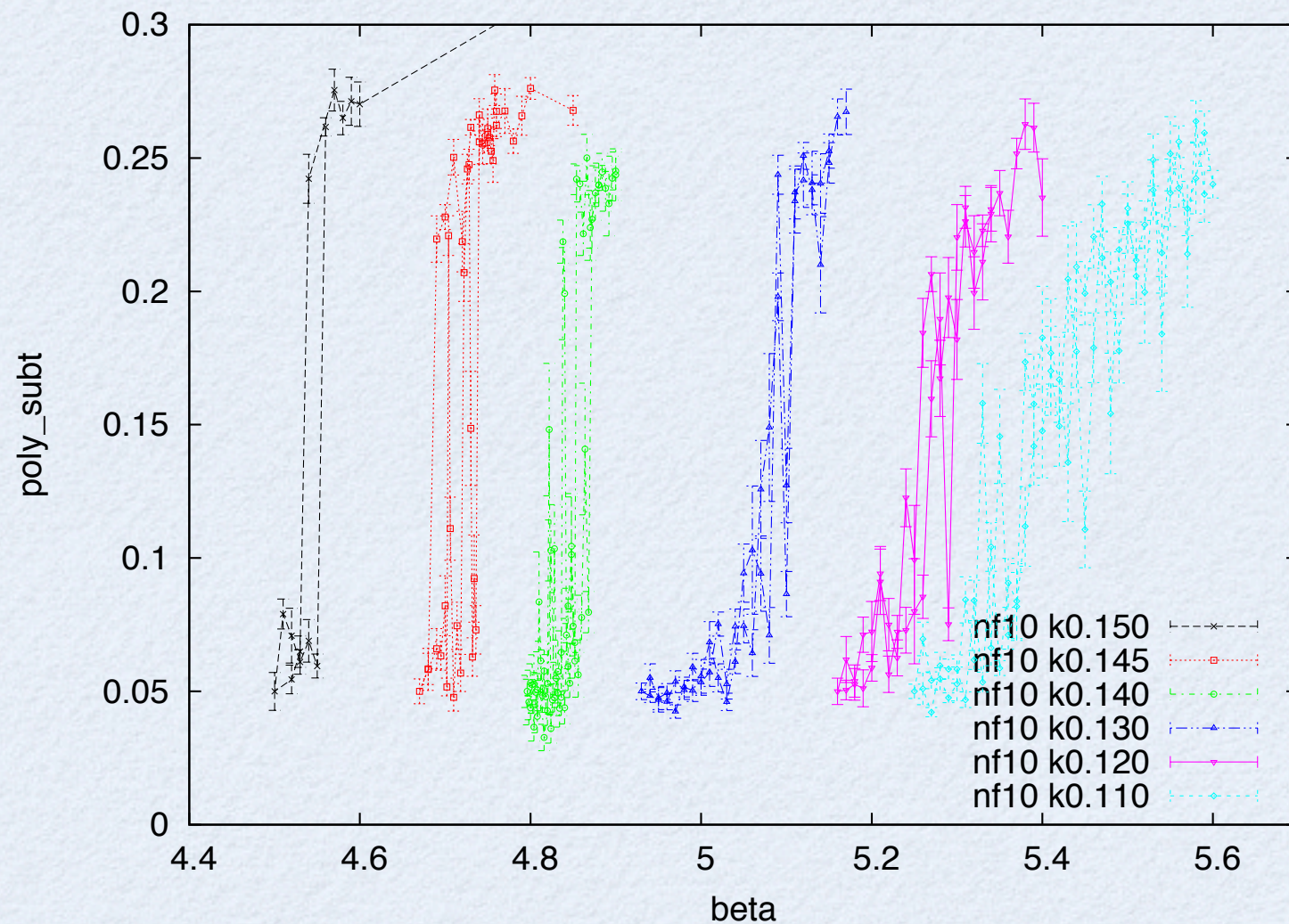
# General Strategy

- ✓ Find the critical endpoint around  $m_q=0$ .
- ✓ Keep monitoring it while changing  $V$  to see whether it moves to the right with  $N_T$  or  $V$ .
- ✓ If the endpoint moves to the right as in QCD, the theory is outside of Conformal Window.



# This work ( $N_F=10$ )

$m_Q$  and  $\beta$  dependence of Polyakov loop@ $N_F=10$



This work using Wilson Fermion on  $V=4^3 \times 4$ .

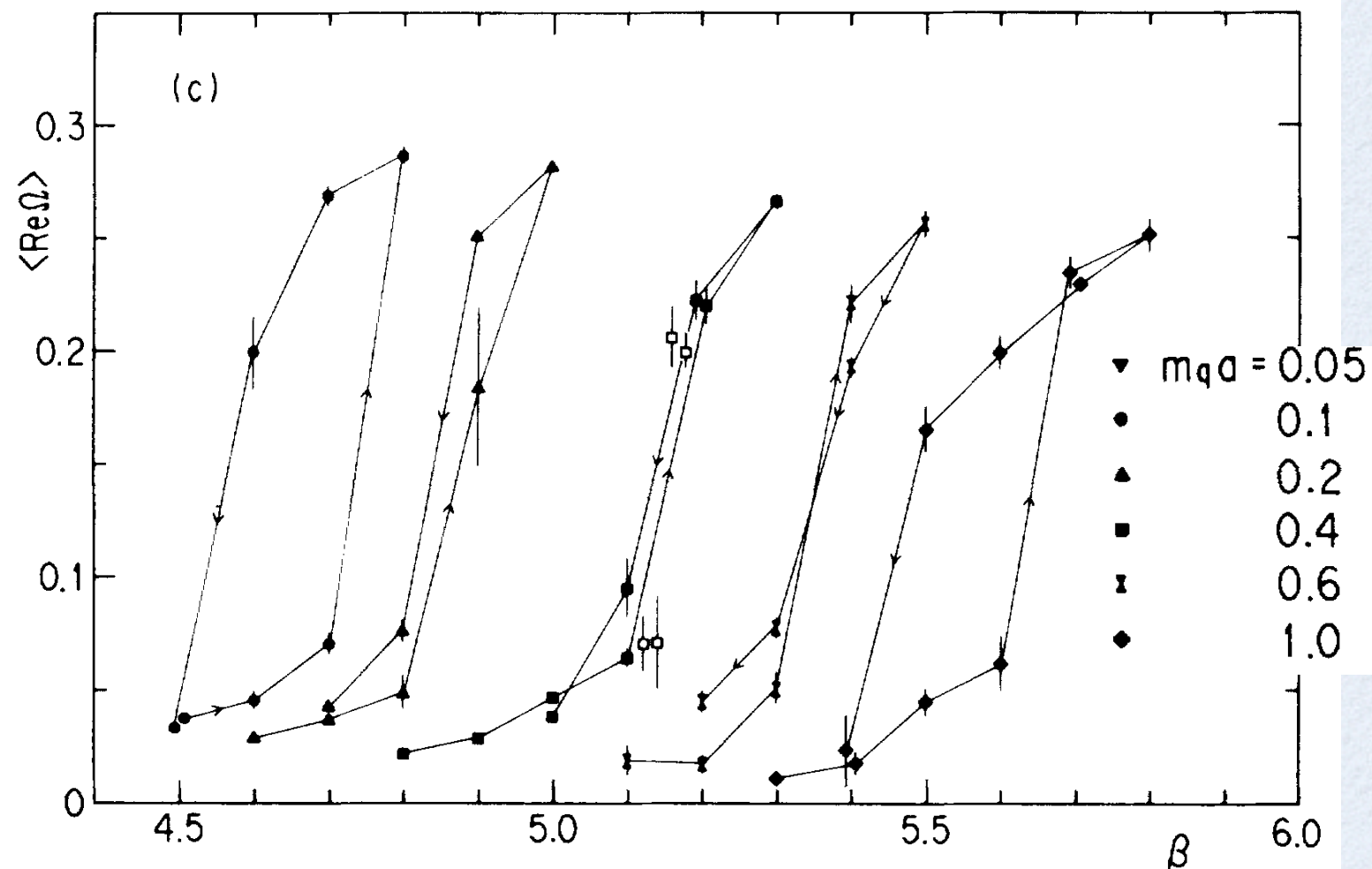
**1st order weakens as going to small kappa.**



# Previous Results ( $N_F=10$ )

Fukugita, Ohta, Ukawa (88)

$m_Q$  and  $\beta$  dependence of Polyakov loop @  $N_F=10$



KS,  $V=8^3 \times 4$ .

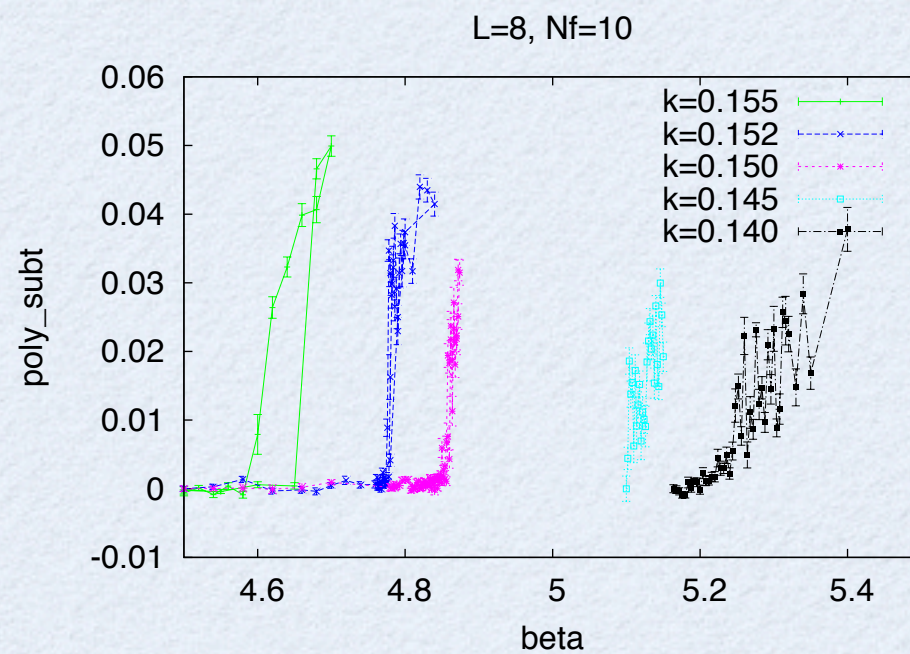
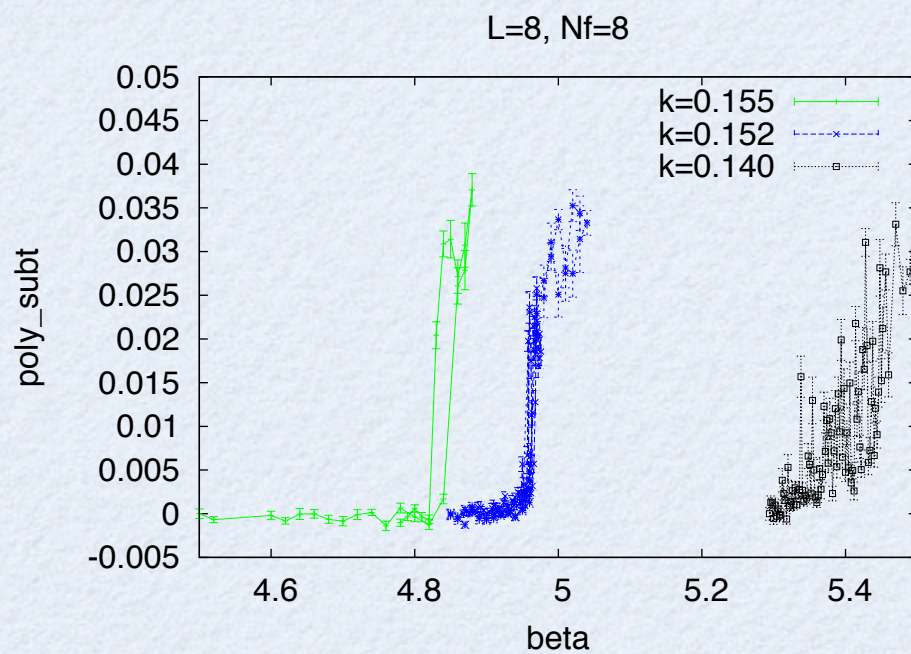
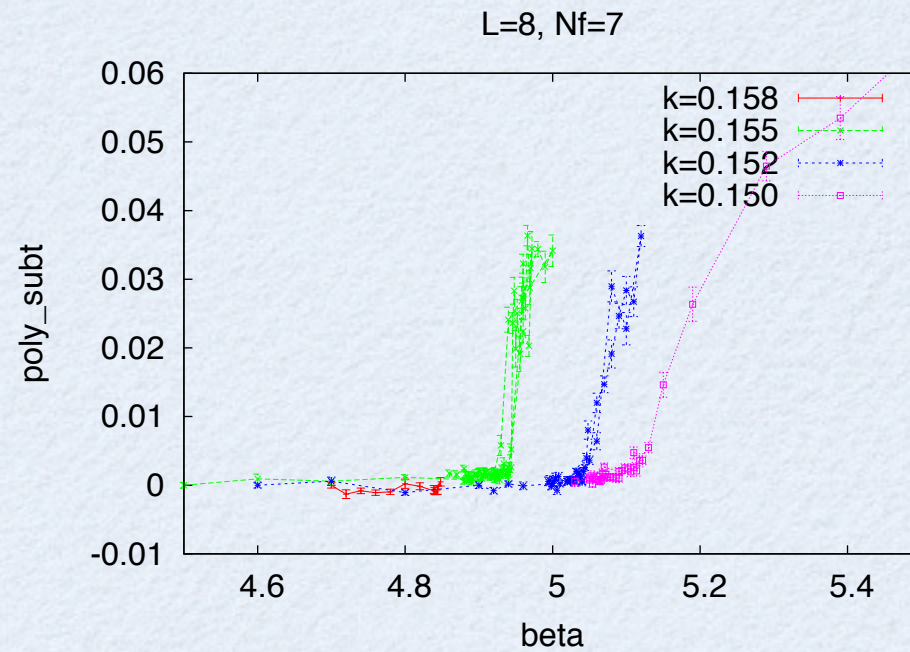
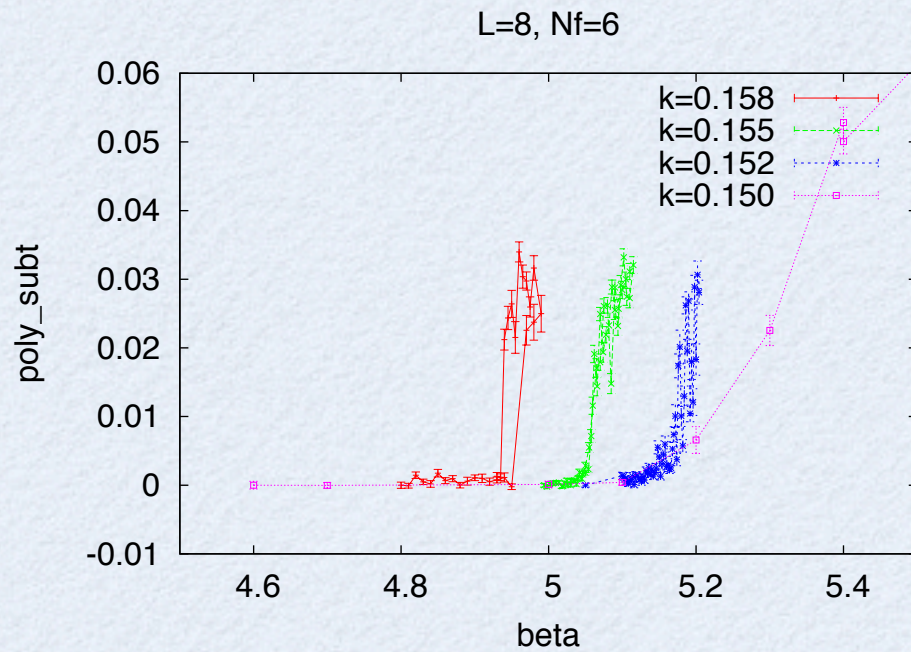
**Observation:**

Transitions are 1st order  
over the entire range of  
fermion mass at  $N_F=10$ .  
No end point.



# $V=8^3 \times 8$ , $N_F=6-10$

## Preliminary Results for $N_F=6, 7, 8, 10$



**1st order at large  
kappa weakens as  
going to small  
kappa.**

**Calc. on  $V=16^4$  is  
on going.**



# (Far) Future plan

(strongly depends on what LHC@CERN observes.)

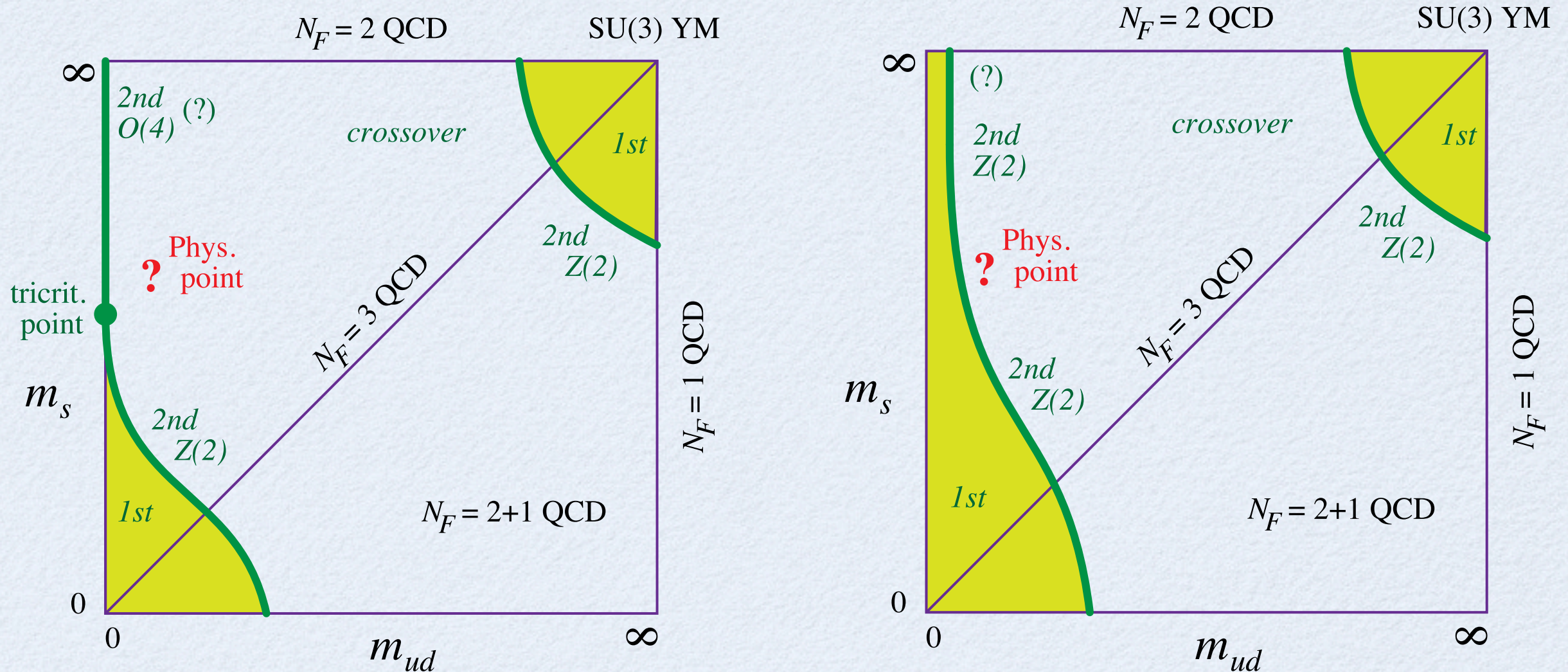
- ✓ In realistic TC model, only two of  $N_F$  flavors must be exact massless while the other  $N_F-2$  flavors shouldn't be so.
- ✓ Go to  $2 + (N_F-2)$  QCD
- ✓ Nice to discuss such a theory on the basis of Columbia plot



# Columbia plot

Brown, Butler, Chen, Christ, Dong, Schaffer, Unger, and Vaccarino (90),  
N.H. Christ, Z. Dong (92) and N.H. Christ(92)

**Kanaya, Lattice 2010**

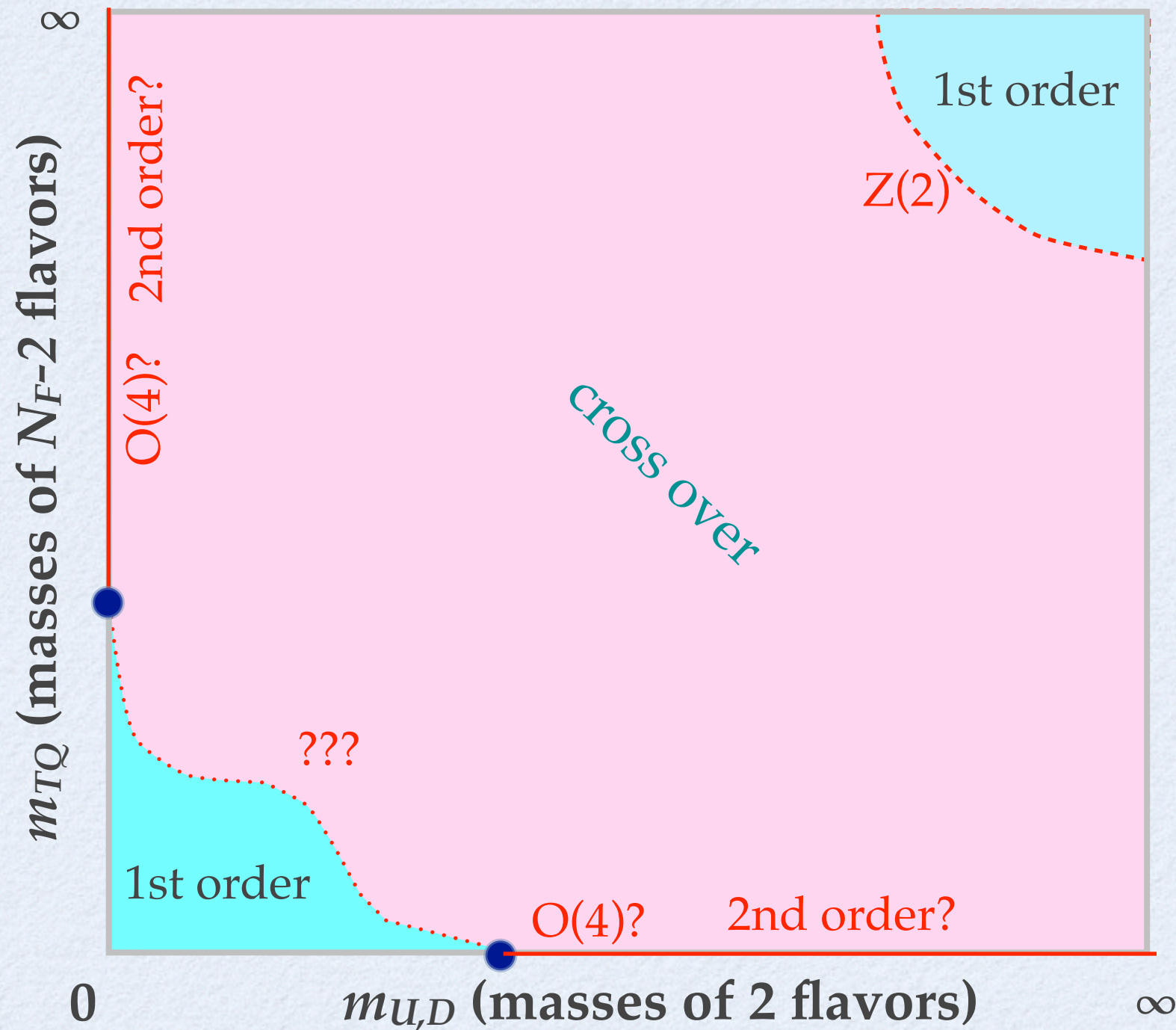


Recent lattice results seems to favor the left.

How does this plot for many flavor QCD look like?



$$N_F = 4 (< N_F^{\text{crit}})$$



$N_F^{\text{crit}} > 4$  is assumed.

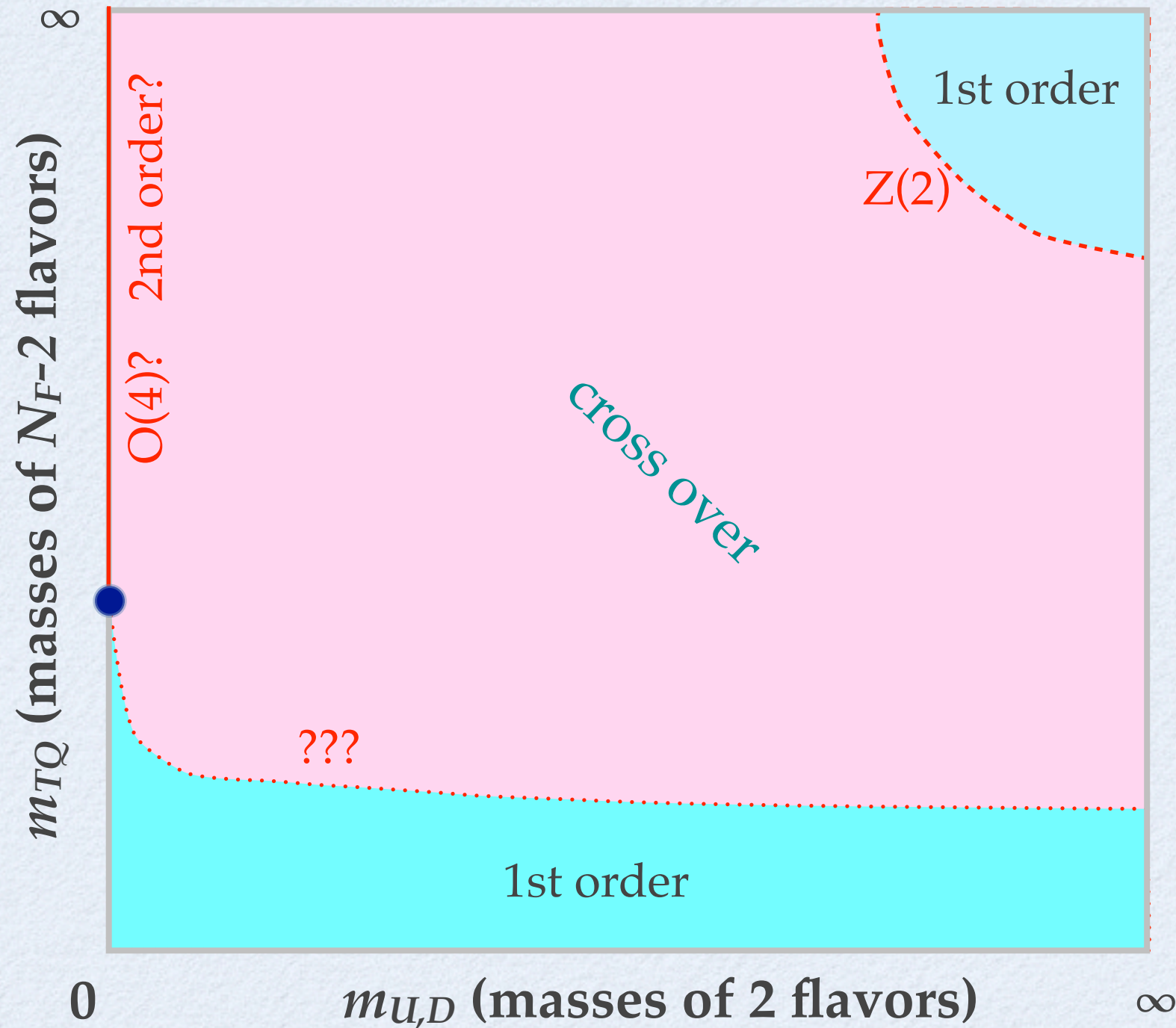
Symmetric phase diagram

Probably running is not slow enough.

Less interesting.



$$5 \leq N_F < N_F^{\text{crit}}$$



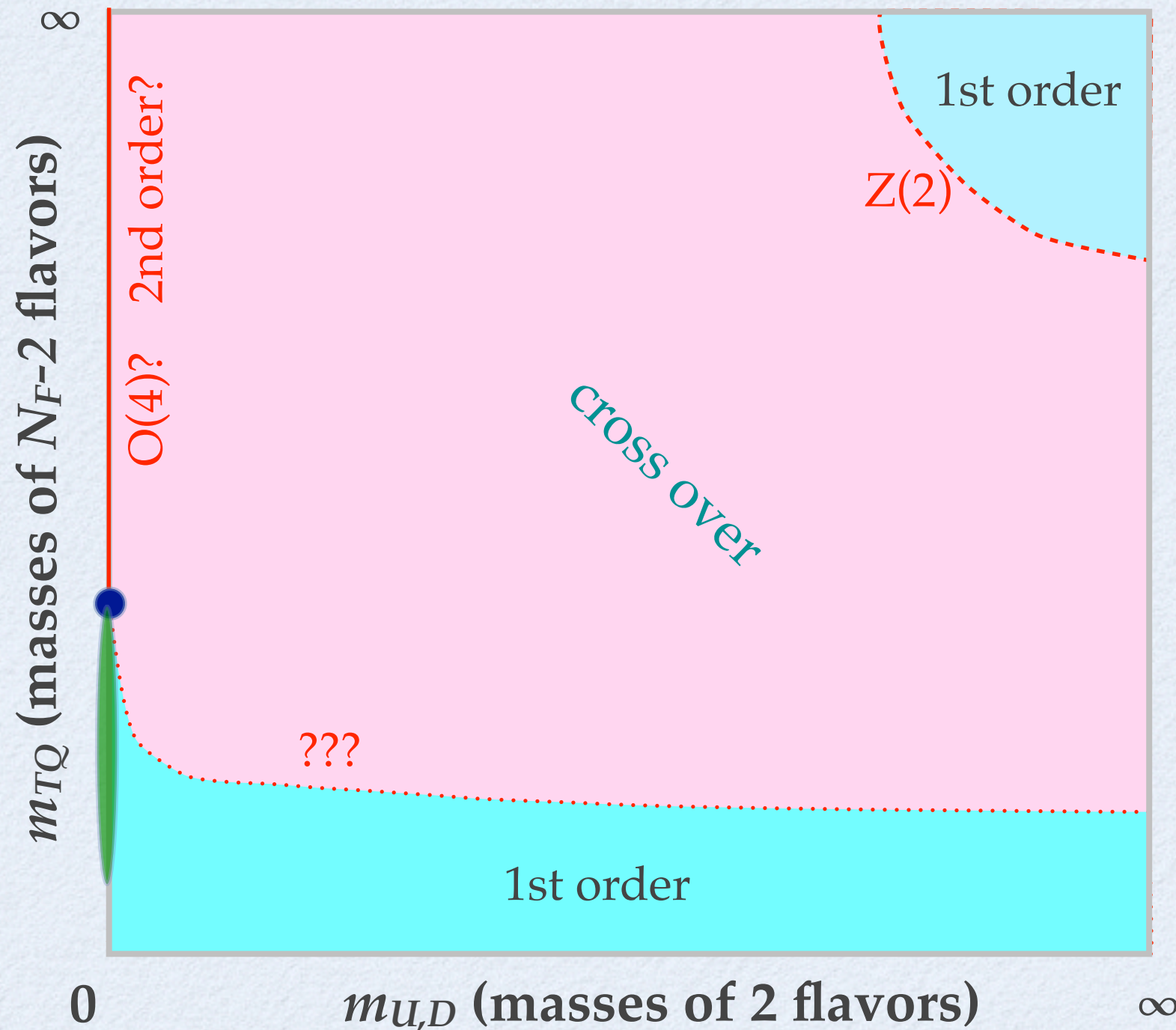
$N_F^{\text{crit}} > 5$  is assumed.

1st order persists to  $m_{U,D} = \infty$  for small  $m_{TQ}$ .

Slow running and large  $\gamma_m$  may be expected at some  $N_F$ .



$$5 \leq N_F < N_F^{\text{crit}}$$



$N_F^{\text{crit}} > 5$  is assumed.

1st order persists to  $m_{U,D} = \infty$  for small  $m_{TQ}$ .

Slow running and large  $\gamma_m$  may be expected at some  $N_F$ .

Furthermore,

- 1st order P.T. is attractive because of baryogenesis.

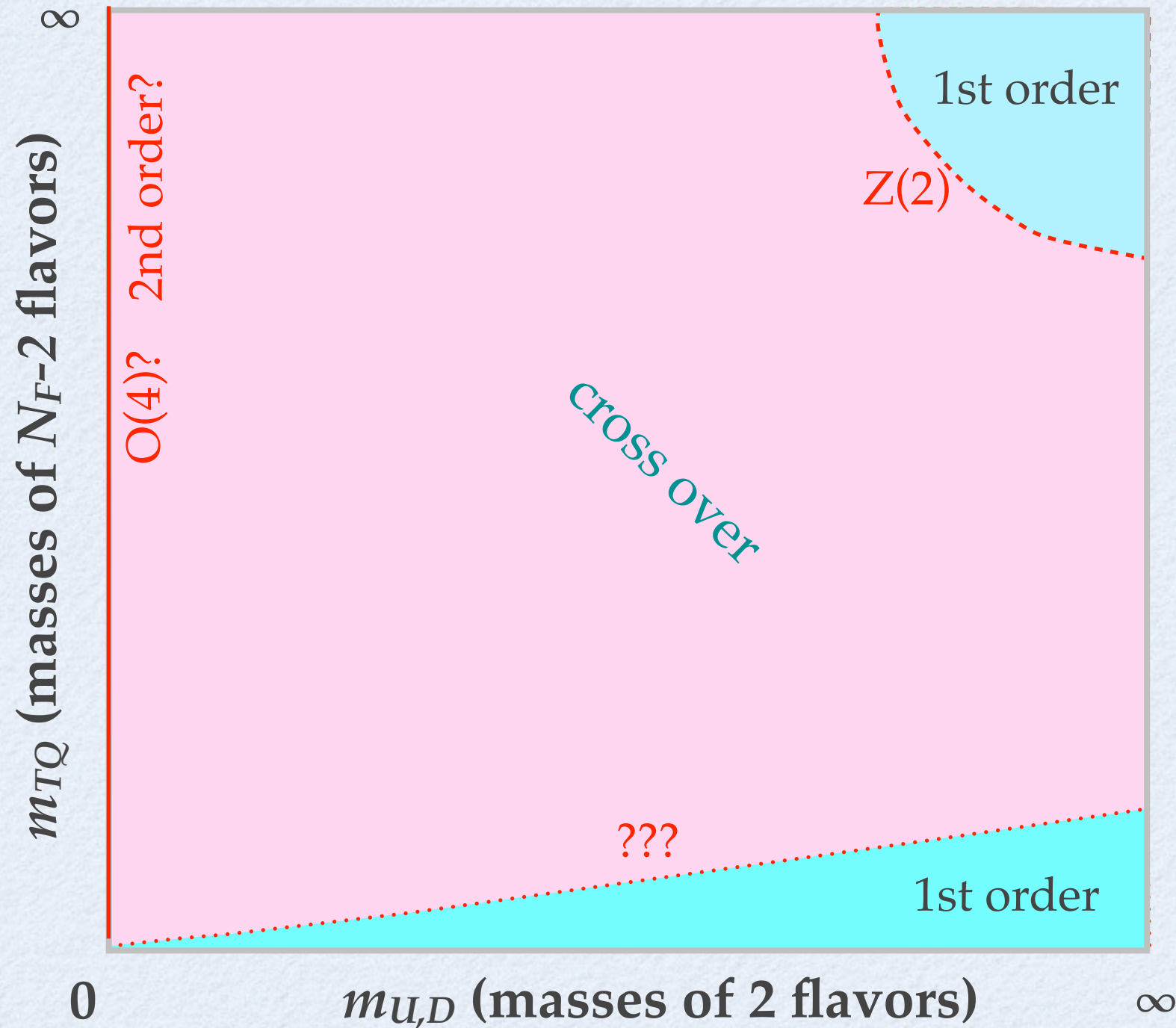
Appelquist, Schwetz and Selipsky, PRD52, 4741 (1995).

Kikukawa, Kohda and Yasuda, PRD77 (2008) 015014

Phenomenologically interesting!



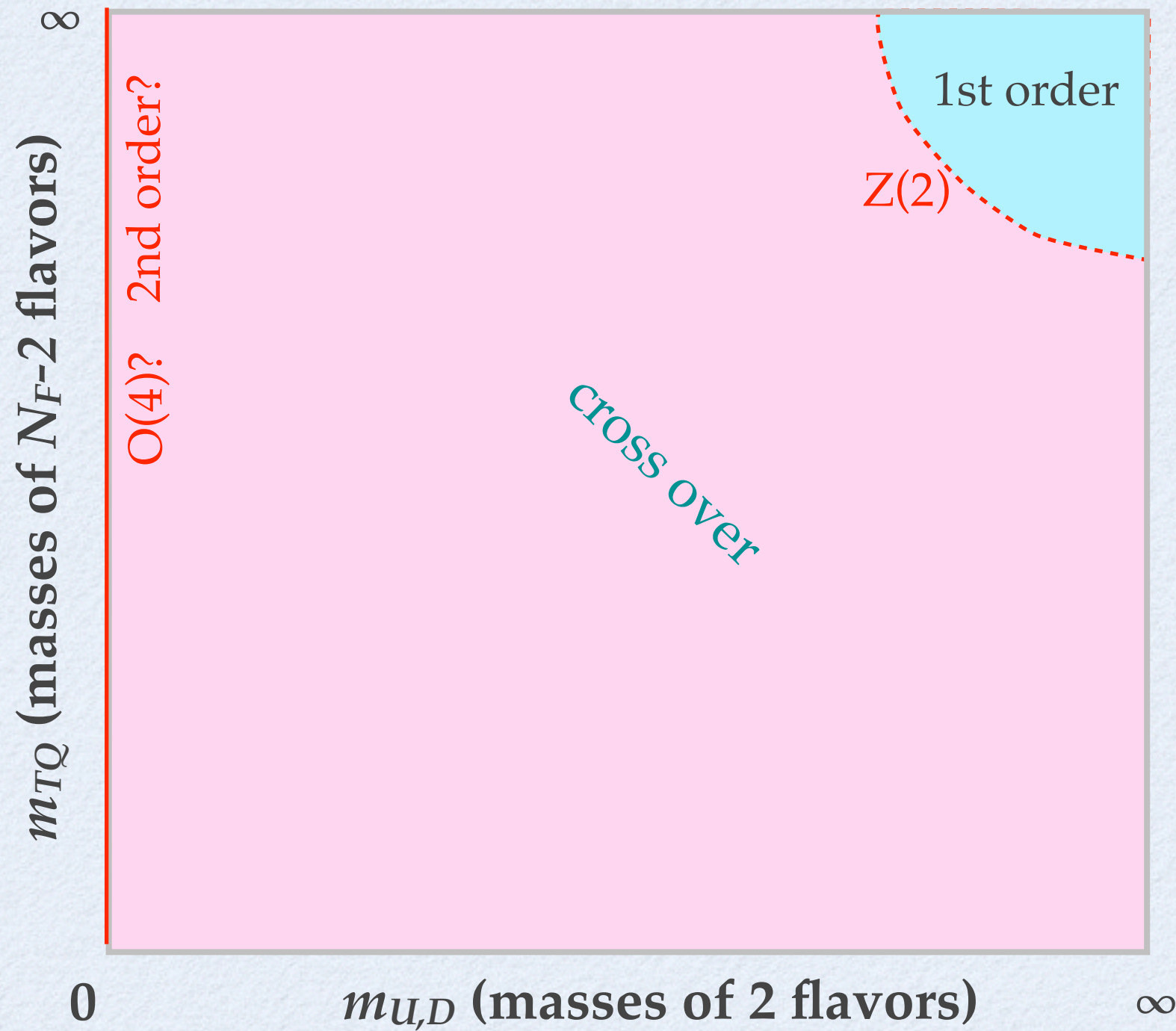
$$N_F - 2 < N_F^{\text{crit}} < N_F \text{ (Speculation)}$$



If this is the case,  
EW Baryogenesis within  
TC seems to difficult.



$$N_F^{\text{crit}} < N_F - 2 < N_F \text{ (Speculation)}$$



If this is the case, not interesting.

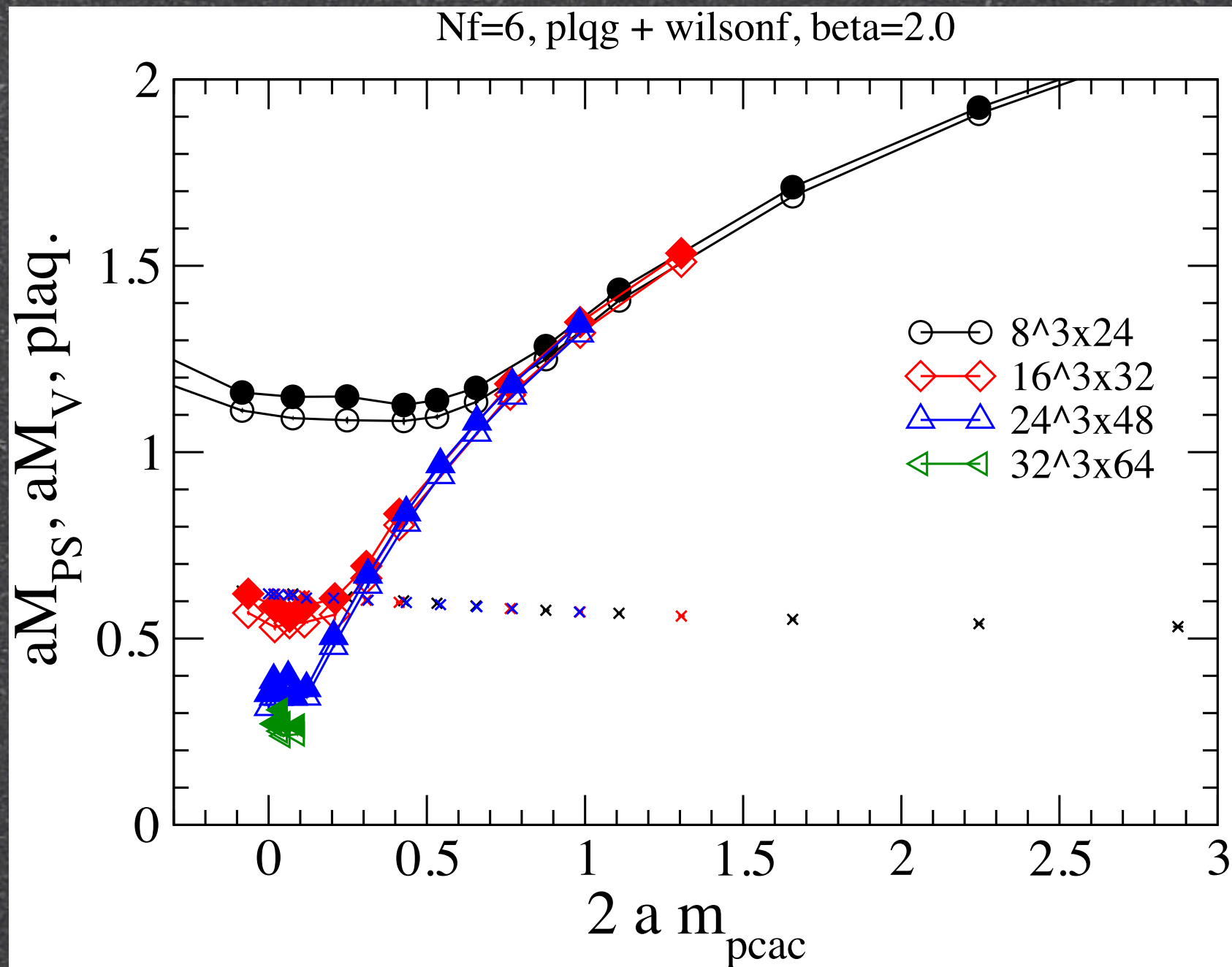


# Summary

- ✓ Important to have several complementary approaches in the search for Conformal Window.
- ✓ We employ Wilson fermion to study the properties of Many Flavor QCD.
- ✓ Important to know Wilson Phase diagram when interpreting spectroscopy results.
- ✓ Establishing Columbia plot for Many Flavor QCD clarifies phenomenologically interesting region.



# Finite Volume effect

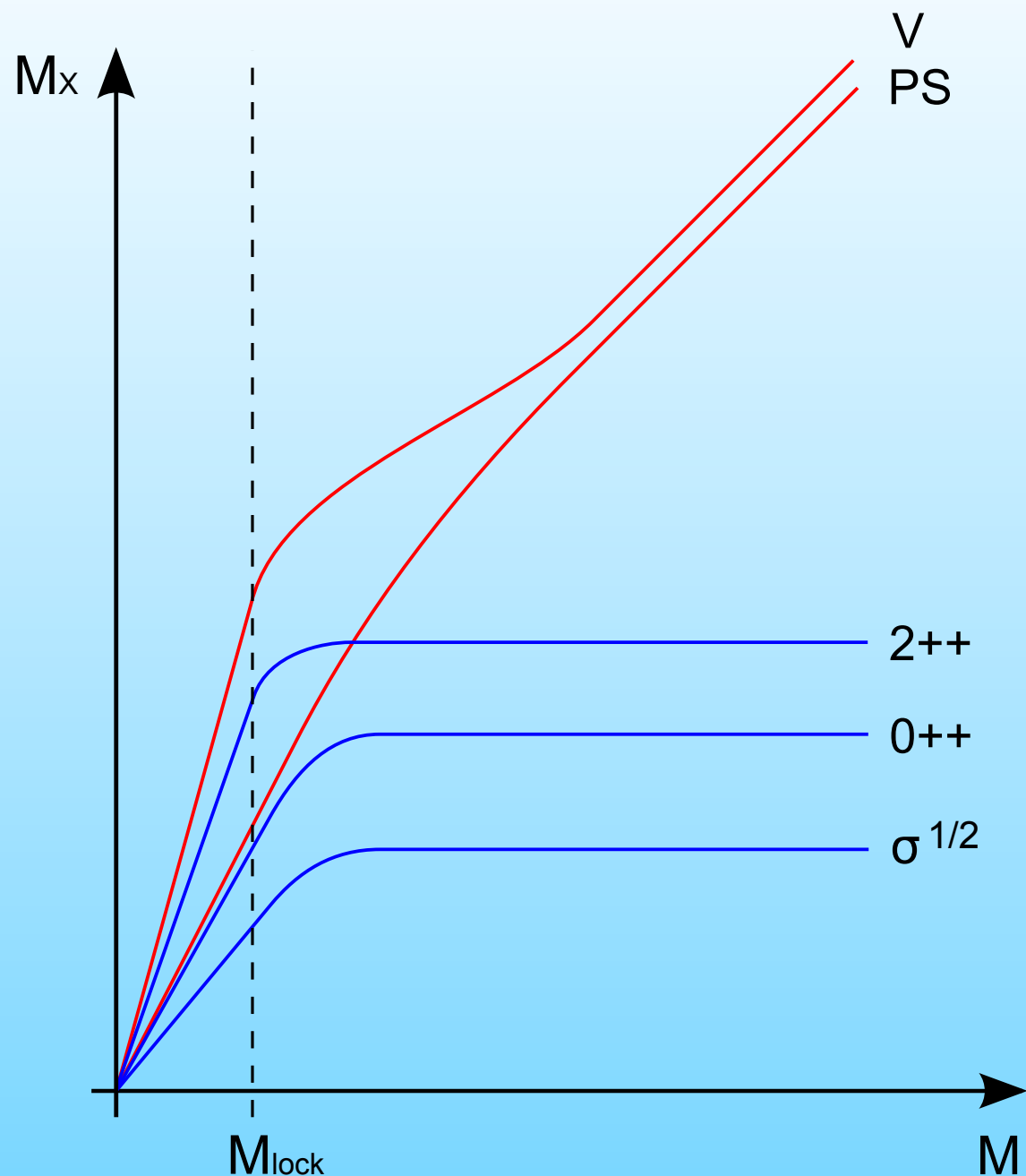


- Finite volume effect is significant.
- Masses are bounded from below.
- Minimum decreases as volume  $\rightarrow$  large.



# Expected behavior in Conformal Window

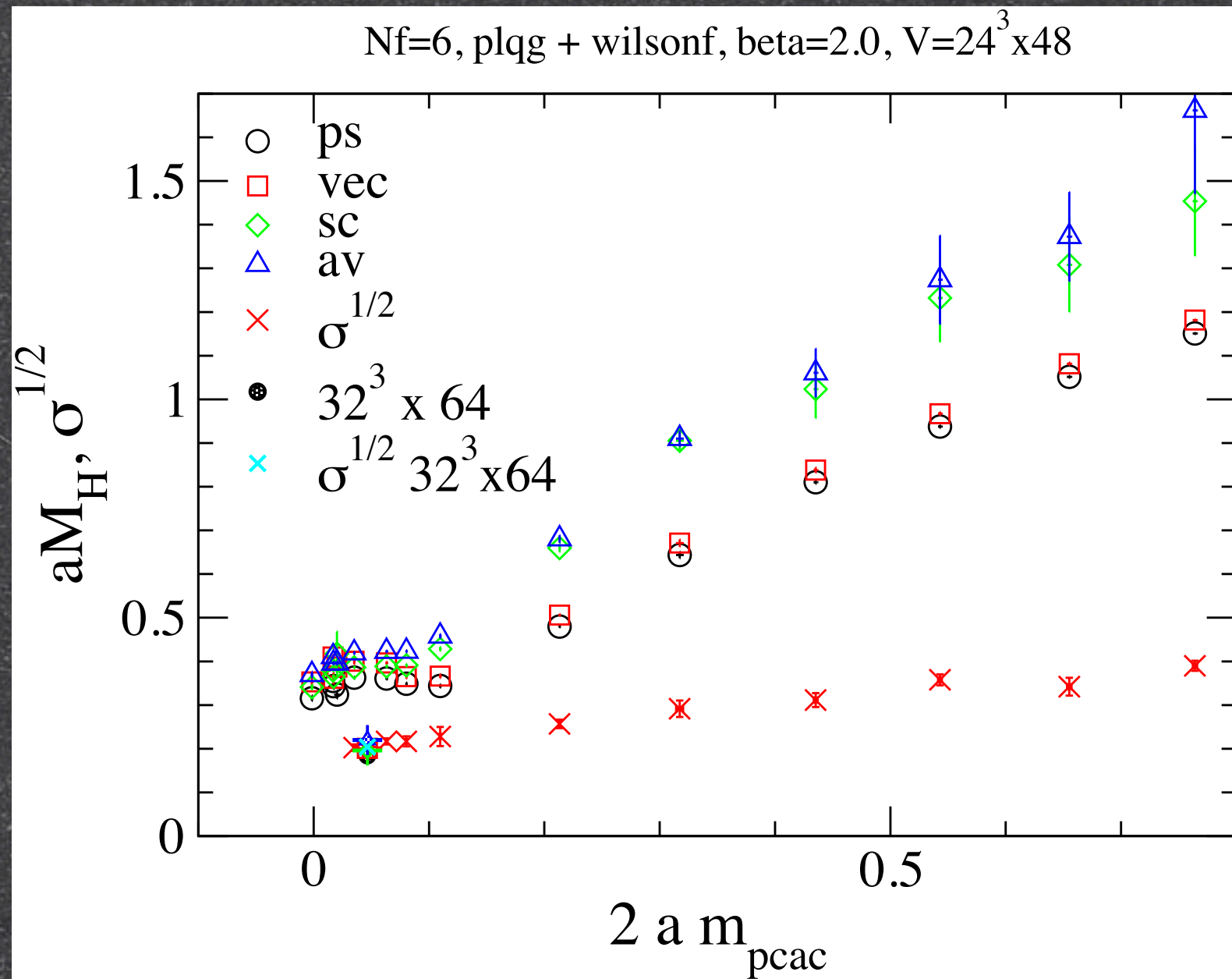
Del Debbio et al.(2010)



- Static limit = Quench
- In the massless limit, everything becomes massless.
- Dynamical scale (e.g.  $\Lambda_{\text{QCD}}$  in QCD) also vanishes there in contrast to QCD.
- Therefore, mass dependence of gluonic quantities is the key.



# $M_H$ and $\sigma$



- $M_P \approx M_V$  and  $M_S \approx M_{AV}$  are typical pattern in the presence of heavy quark symmetry.
- $\sigma^{1/2}$  is smaller than  $M_H$  in most region.
- At  $V=32^3 \times 64$ ,  $\sigma^{1/2} \approx M_H$
- FVE is small for  $\sigma^{1/2}$ .
- $\sigma^{1/2}$  seems to remain finite in the chiral limit.
- Confinement?